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Local quasi hidden variable modelling and violations of Bell-type inequalities by a multipartite quantum state

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We introduce for a general correlation scenario a new simulation model, *a local quasi hidden variable* (LqHV) *model*, where locality and the measure-theoretic construction inherent to a local hidden variable (LHV) model are preserved but positivity of a simulation measure is dropped. We specify a necessary and sufficient condition for LqHV modelling and, based on this, prove that every quantum correlation scenario admits a LqHV simulation. Via the LqHV approach, we construct analogs of Bell-type inequalities for an N-partite quantum state and find a new analytical upper bound on the maximal violation by an N-partite quantum state of $S_1 \times \cdots \times S_N$-setting Bell-type inequalities – either on correlation functions or on joint probabilities and for outcomes of an arbitrary spectral type, discrete or continuous. This general analytical upper bound is expressed in terms of the new state dilation characteristics introduced in the present paper and not only traces quantum states admitting an $S_1 \times \cdots \times S_N$-setting LHV description but also leads to the new exact numerical upper estimates on the maximal Bell violations for concrete N-partite quantum states used in quantum information processing and for an arbitrary N-partite quantum state. We, in particular, prove that violation by an N-partite quantum state of an arbitrary Bell-type inequality (either on correlation functions or on joint probabilities) for S settings per site cannot exceed $(2S - 1)^{N-1}$ even in case of an infinite dimensional quantum state and infinitely many outcomes. © *2012 American Institute of Physics*. [http://dx.doi.org/10.1063/1.3681905]

I. INTRODUCTION

The seminal papers of Einstein, Podolsky, and Rosen[1] (EPR) and Bell[2,3] are still the ones most cited in quantum information. In Ref. 1, Einstein *et al.* argued that *locality* of measurements performed by spatially separated parties on perfectly correlated quantum events implies the "simultaneous reality – and thus definite values" of physical quantities described by noncommuting quantum observables. Based on this argument contradicting, however, the quantum formalism and referred to as the EPR paradox, Einstein *et al.* expressed in Ref. 1 their belief on a possibility of a *hidden variable* account of quantum measurements.

Analyzing this EPR belief in 1964–1966, Bell explicitly constructed[2] the hidden variable model reproducing the statistical properties of all quantum observables for a qubit but, however, proved[3] that, for bipartite measurements on a two-qubit system in the singlet state, a *local* hidden variable description (LHV) disagrees with the statistical predictions of quantum theory. Based on these results, Bell concluded[2] that the EPR paradox should be resolved specifically via violation of *locality* under bipartite quantum measurements and that "...non-locality is deeply rooted in quantum mechanics itself and will persist in any completion."

Ever since 1964, the conceptual and mathematical aspects of the probabilistic description of multipartite quantum measurements have been analyzed in a plenty of papers, see, for example,

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